Exercise 86

In Section 1.4 we modeled the world population from 1900 to 2010 with the exponential function

$$P(t) = (1436.53) \cdot (1.01395)^t$$

where $t = 0$ corresponds to the year 1900 and $P(t)$ is measured in millions. According to this model, what was the rate of increase of world population in 1920? In 1950? In 2000?

Solution

The rate of increase of the world population is given by the derivative of $P(t)$.

$$\begin{aligned}
P'(t) &= \frac{d}{dt}[P(t)] \\
&= \frac{d}{dt}[(1436.53) \cdot (1.01395)^t] \\
&= 1436.53 \frac{d}{dt}[(1.01395)^t] \\
&= 1436.53 \frac{d}{dt}\left[e^{\ln(1.01395)^t}\right] \\
&= 1436.53 \frac{d}{dt}\left[e^{t\ln(1.01395)}\right] \\
&= 1436.53 \left\{e^{t\ln(1.01395)} \cdot \frac{d}{dt}[t\ln(1.01395)]\right\} \\
&= 1436.53 \left\{e^{t\ln(1.01395)} \cdot [\ln(1.01395)]\right\} \\
&= 1436.53 \ln(1.01395) e^{t\ln(1.01395)} \\
&= 1436.53 \ln(1.01395)(1.01395)^t
\end{aligned}$$

Evaluate it at $t = 20$ and $t = 50$ and $t = 100$.

$$\begin{aligned}
P'(20) &\approx 26.25 \text{ million/year} && \text{(Rate of population growth in 1920)} \\
P'(50) &\approx 39.78 \text{ million/year} && \text{(Rate of population growth in 1950)} \\
P'(100) &\approx 79.53 \text{ million/year} && \text{(Rate of population growth in 2000)}
\end{aligned}$$